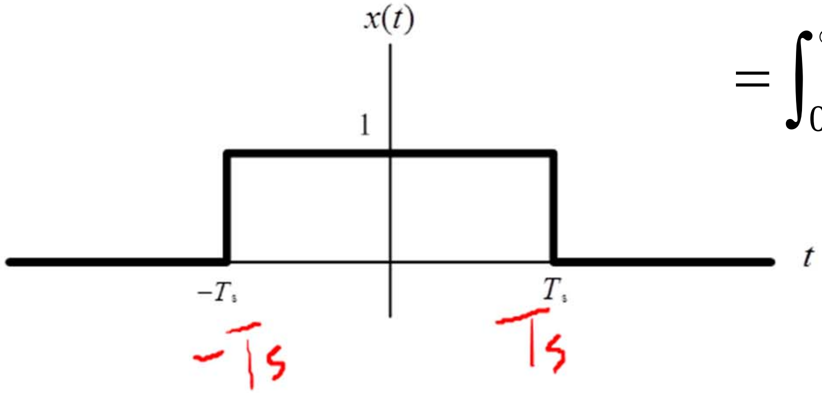
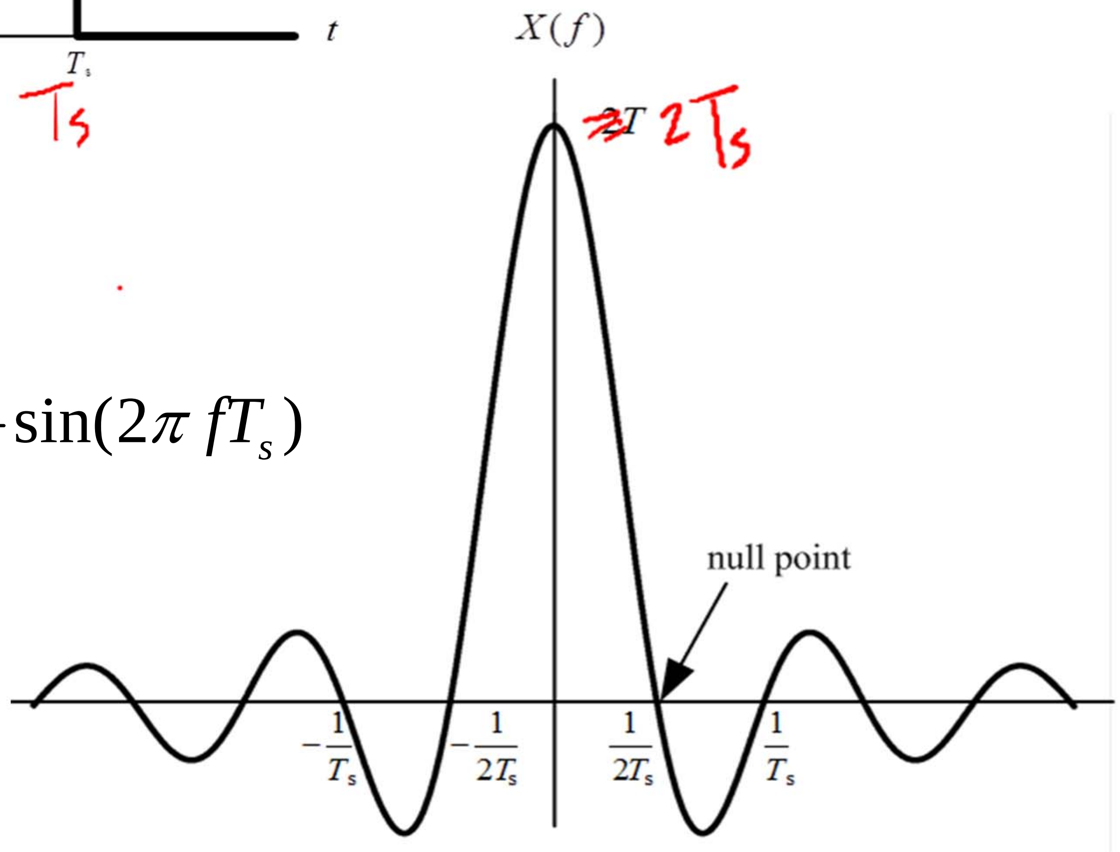
Example 1:

(A) Fourier Transform

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df$$

$$= \int_{0}^{\infty} \frac{2}{\pi f} \sin(2\pi f T_s) \cos(2\pi f t) df$$





$$X(-f) = X(f) = \frac{1}{\pi f} \sin(2\pi f T_s)$$

$$x(t)=\int_{-\infty}^{\infty}\frac{1}{\pi f}\sin(2\pi f T_s)e^{j2\pi f t}df \quad (1)$$

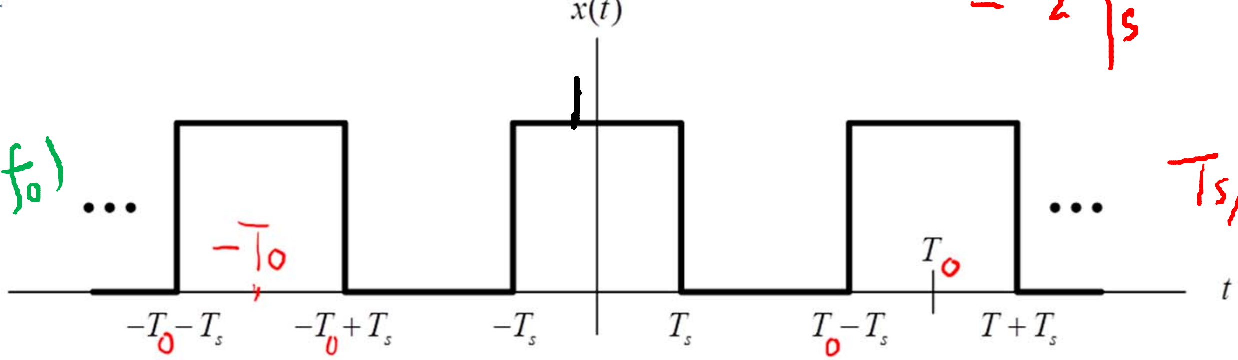
① $f = kf_0$

$f_0 = \frac{1}{T_0}$

② $\lim_{f\to 0}\frac{\sin(2\pi f T_s)\cos(2\pi f t)}{\pi f} = 2T_s$

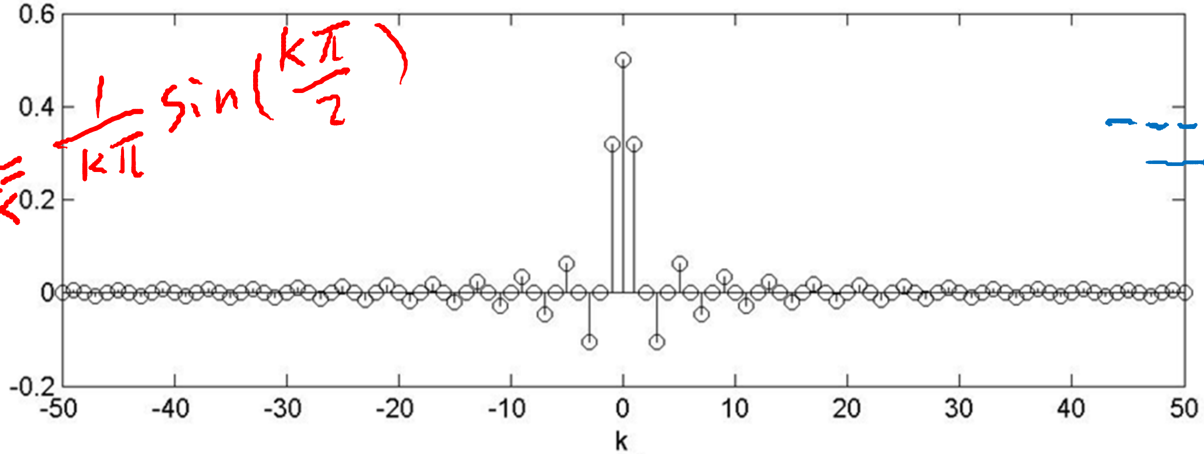
(B) Fourier Series

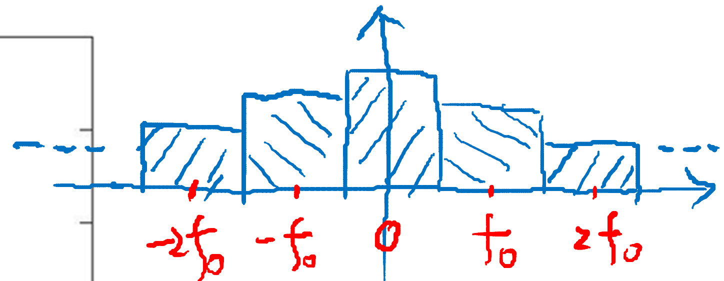
$X_k = \frac{1}{T_0}X(kf_0)$



$T_s/T_0 = \frac{1}{4}$

$X_k = \frac{1}{k\pi}\sin(\frac{k\pi}{2})$





$$X_{-k}=X_k=\frac{1}{\pi k}\sin(\frac{2\pi k T_s}{T_0})$$

$$|e^{j2\pi k f_0 t}|=1$$

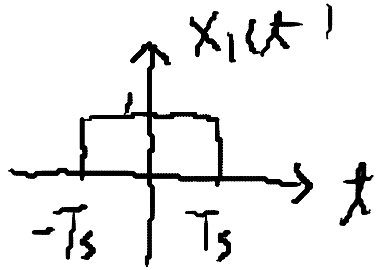
$$x(t)=\sum_{k=-\infty}^{\infty}X_k e^{j2\pi k f_0 t}=2T_s\times f_0+\sum_{k=-\infty}^{-1}\frac{1}{\pi k f_0}\sin(2\pi k f_0 T_s)e^{j2\pi k f_0 t}f_0$$

$$+\sum_{k=1}^{\infty}\frac{1}{\pi k f_0}\sin(2\pi k f_0 T_s)e^{j2\pi k f_0 t}f_0=\sum_{k=-\infty}^{\infty}|X(kf_0)|\times f_0=\frac{2T_s}{T_0}+\sum_{k=1}^{\infty}\frac{2}{\pi k}\sin(\frac{2\pi k T_s}{T_0})\cos(2\pi k f_0 t). \quad (2)$$

$$\left|\frac{1}{\pi k f_0}\sin(2\pi k f_0 T_s)e^{j2\pi k f_0 t}\right| = \left|\frac{1}{\pi k f_0}\sin(2\pi k f_0 T_s)\right| = \left|X(kf_0)\right| \times f_0\,(底)$$

高 = 面積

$$e^{j2\pi k f_0 t} + e^{-j2\pi k f_0 t} = \cos(2\pi k f_0 t)$$



- Fourier transform (1) → Fourier series (2)

$kf_0 \to f;\ f_0 \to df$

$T_0 \to \infty$

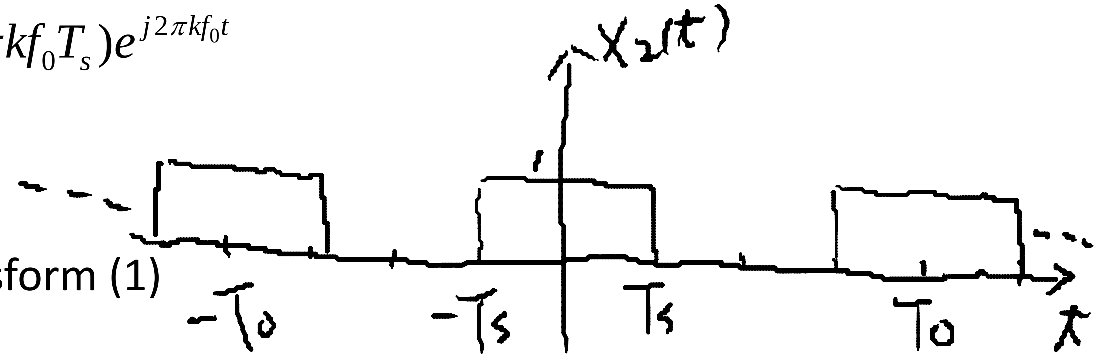
$$x_1(t) = \int_{-\infty}^{\infty} \frac{1}{\pi f}\sin(2\pi f T_s)e^{j2\pi f t}\,df \quad (1)$$

$f \to kf_0$

$df \to f_0$

$$x_2(t) = 2T_s \times f_0 + \sum_{k=-\infty}^{-1} \frac{1}{\pi k f_0}\sin(2\pi k f_0 T_s)e^{j2\pi k f_0 t} f_0 + \sum_{k=1}^{\infty} \frac{1}{\pi k f_0}\sin(2\pi k f_0 T_s)e^{j2\pi k f_0 t} f_0 \quad (2)$$

$$= \sum_{k=-\infty}^{\infty} X_k e^{j2\pi k f_0 t} = \sum_{k=-\infty}^{\infty} \frac{1}{\pi k}\sin(2\pi k f_0 T_s)e^{j2\pi k f_0 t}$$



- Fourier series (2) → Fourier transform (1)

$T_0 = 4T_s$

Example 2:
(exercise)

$$x(t) = \sum_{n=1}^{\infty} \frac{2(1-\cos(\pi n))}{\pi n} \sin(2\pi n f_0 t).$$