資工系 **Error Control Coding　期末考　　99. 1.19 (open book)**

1. (30%) Consider the Galois field $GF(4^2)$.

   Find a (15,9) double-error-correcting BCH code over GF(4).

   (1) $g(x) = ?$ (take $j_0 = 3$)

   (2) Which roots does the g(x) have over $GF(4^2)$?

   (3) Suppose the input information polynomial $i(x)=2+3x^4$, the error pattern polynomial $e(x)= 3x^2$. Find the codeword polynomial c(x) and the received polynomial r(x)?

   (4) Devise a Peterson-Gorenstein-Zierler decoder for this code.

2. (20%) Find the generator polynomials and the minimum distance for the following codes：(primitive polynomial x^4+x+1, x^5+x^2+1 over GF(2))

   (a) RS(15,7) code

   (b) RS(31,21) code

3. (25%)

   (a) Find a (7, 3) Reed-Solomon code over $GF(2^3)$.

   (1) $g(x) = ?$ (take $j_0 = 3$)

   (2) Which roots does the g(x) have over $GF(2^3)$?

   (3) What is the codeword for the input information vector $(\alpha^3,0,0)$, where $\alpha^3 \in GF(2^3)$.

   (b) Devise a Peterson-Gorenstein-Zierler decoder for the received vector $(1, \alpha^3, 1, \alpha, \alpha^3, 0, \alpha^3)$.

   (c) Which bit(s) is (are) erroneous ?

4. (25%) Consider the Galois field $GF(2^3)$. The element $\beta = \alpha^3$ is a primitive element. Let $g_0(x)$ be the lowest-degree polynomial over GF(2) which has

   β, β^2

   as its roots. This polynomial can generate a one-error-correcting primitive BCH code of length 7.

   (a) Find the parity-check matrix [each element is belong to GF(2)] for this code by the method shown in pp. 13 of lecture note BCH_Code.

   (b) Determine g(x).

   (c) Find the parity-check matrix for this code by the method shown in pp. 10 of lecture note Cyclic_Code.