資工系 **Error Control Coding 期中考 99. 11.10 (open book)**

1. (10%) An n-tuple v is a codeword in the code generated by G iff $v \cdot H^T = 0$.
2. (20%) Consider a systematic (8,4) code whose parity-check equations are

$$v_0 = u_1 + u_2 + u_3$$
$$v_1 = u_0 + u_1 + u_2$$
$$v_2 = u_0 + u_1 + u_3$$
$$v_3 = u_0 + u_2 + u_3$$

where u_0, u_1, u_2, u_3 are message digits and v_0, v_1, v_2, v_3 are parity-check digits. Find the generator and parity-check matrices for this code. Show analytically that the minimum distance of this code is 4.

3. (10%) In the lecture note (pp. Linear block code 17). Suppose the transmitted codeword C=(110110) and the channel error pattern E=(100010). Explain that the received vector (=C+E) can't be decoded correctly by the standard array method.

4. (10%) Explain that why $(GF(2)[x], \oplus, \otimes)$ is a ring and not a field?

5. (10%) For the Galois Field $GF(4^2)$ which was shown in the lecture note (pp. abstract algebra 19). Find the value of the operation $(2Z+3) \bullet (3Z+2)$.

6. (10%) Prove that the constant coefficient g_0 of the generator polynomial $g(x) = g_0 + g_1x + \cdots + g_{n-k-1}x^{n-k-1} + x^{n-k}$ over GF(2) is equal to one.

7. (10%) Find the minimal polynomial over GF(2) of α^6 in $GF(2^4)$.

8. (20%)
   (a) (10%) Show that $g(x)= 1+x+x^4$ generates a (15,11) cyclic Hamming code.
   (b) (10%) Devise an encoder for the (15,11) cyclic Hamming code.

9. (10%) Describe the relationship between s(x) and rH^T.