Graph Theory (圖型理論)

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Introduction

- 圖論為一在二十世紀快速成長的一門學問。自原本只是組合數學書中一個章節的短短數頁，百倍增長為如今此一可單獨研討的學門。它之所以會如此快訴成長，主要的原因是圖論的應用性非常之廣。諸如：物理、化學、生物、心理學、社會學等等。而另一個原因則是，在電腦科學理論上，一些關於處理複雜度的問題，能被轉換成圖論問題來解決。

Introduction

- 師生晤談時間： 二EF
- 主要教科書：Junming Xu, Theory and Application of Graphs, Kluwer Academic Publishers (科大文化代理), 2003.
- 重要參考書籍：

1. West, Introduction to Graph Theory 2ed, Prentice-Hill (全華代理), 2001.
2. Bondy and Murty, Graph Theory with Application. 1976.
3. Balakrishnan and Ranganatha, A Textbook of Graph Theory 2ed, Springer, 2012.
4. 張鎮華,演算法觀點的圖論, 台灣大學出版社, 2017.

Introduction

- 課程內容： 預計將介紹下列各項

1. Basic Concepts of Graphs
2. Trees and Graphic Spaces
3. Plans Graphs and Planar Graphs
4. Flows and Connectivity
5. Matchings and Independent Sets
6. Coloring Theory

Introduction

- 評量方式：最高分99

  作業40% + 平時成績20% + 期中考20% + 期末報告 30% + 加分作業

  大學部及格60, 研究生及格70

- 進度:

  第七週、第十週參加會議，補課。

  11/29 期中考：Chap 1 ~ Chap 2.

  1/3, 1/10：期末報告

- 助教： 王怡君 : 計算理論研究室R307-1 (分機4862)

  Office Hour : 無，但可以隨時傳訊息問問題。

- 網頁: http://www.csie.ncnu.edu.tw/~jsjuan/courses.html

圖型理論課程

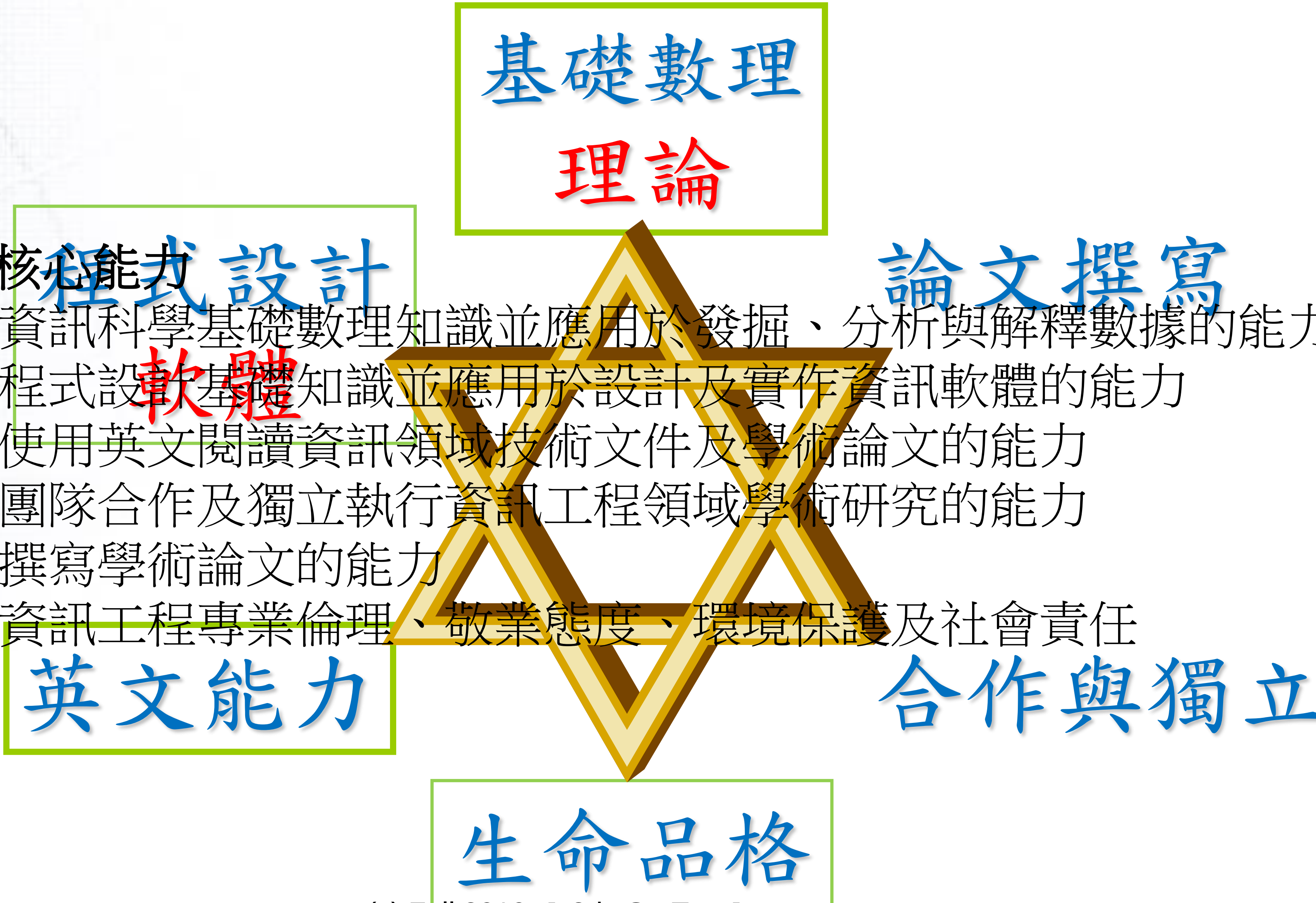
核心能力 與 課程地圖

暨大資工系 教育目標

研究所教育目標

1. 配合國家經濟發展，培養符合資訊產業需求的工程技術人才
2. 配合國家科技及學術發展，培養具備前瞻資訊科技研發能力的人才
3. 配合全球永續發展潮流，培養具備國際視野、工程倫理、人文關懷及社會責任的科技人才

暨大資工系 核心能力



研究所核心能力

1. 具備資訊科學基礎數理知識並應用於發掘、分析與解釋數據的能力
2. 具備程式設計基礎知識並應用於設計及實作資訊軟體的能力
3. 具備使用英文閱讀資訊領域技術文件及學術論文的能力
4. 具備團隊合作及獨立執行資訊工程領域學術研究的能力
5. 具備撰寫學術論文的能力
6. 理解資訊工程專業倫理、敬業態度、環境保護及社會責任

暨大資工系 課程地圖（部分）

暨大資工系課程綜覽-100 學年度（2011/09/13 修訂版）

基礎課程（必修課程）72 學分				專業領域（選修）32 學分
大一	大二	大三	大四	開課年級依各課程決定
普通物理（上）	工程數學	微算機系統		多媒體領域
微積分（上）	邏輯設計	系統程式		演算法與計算理論領域
離散數學	數位電子學	資料庫系統		數位網路通訊領域
計算機概論	資料結構與演算法（一）	專題（一）		訊號與資訊處理領域
普通物理（下）	資料結構與演算法（二）	作業系統		硬體與電路領域
微積分（下）	機率	編譯器		網通技術與文創觀光應用服務領域
程式設計	計算機組織與結構	計算機網路		
電子電路	線性代數	專題（二）		
	數位電路實驗	微算機實驗		

全校共同課程14學分
通識領域課程17學分
專業選修 >= 32學分
未來發展（職涯）

暨大科技學院 核心能力

- 1. 專業知識與實務技能
- 2. 創新與獨立思考能力
- 3. 溝通表達與團隊合作精神
- 4. 專業倫理與社會責任認知
- 5. 掌握國際趨勢與全球視野

暨大學生 八大基本素養與核心能力

- （一）道德思辨與實踐能力
- （二）人際溝通與表達能力
- （三）獨立思考與創新能力
- （四）人文關懷與藝術涵養
- （五）專業知能與數位能力
- （六）團隊合作與樂業倫理
- （七）全球視野與尊重多元文化
- （八）社區參與與公民責任

Chapter 1
Basic Concepts of Graphs

§ 1.1 Graph and Graphical Presentation

1.1 Graph and Graphical Presentation

- **Def:**
  - A **graph G** is an ordered triple **(V, E, ψ)**, where
    V, E: disjoint sets; ψ: $E \to V \times V$: a mapping.
  - V: **vertex-set**; $x \in V$: **vertex**
    E: **edge-set**; $e \in E$: **edge**
    ψ: **incidence function**; if $\psi(e) = (x, y)$: x, y are **end-vertices** of e.

- **Example 1.1.1:**
  $D = (V(D), E(D), \psi_D)$ is a digraph, where
  $V(D) = \{x_1, x_2, x_3, x_4, x_5\}$,
  $E(D) = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9\}$
  and ψ_D is defined by $\psi_D(a_1) = (x_1, x_2)$, $\psi_D(a_2) = (x_3, x_2)$, $\psi_D(a_3) = (x_3, x_3)$,
  $\psi_D(a_4) = (x_4, x_3)$, $\psi_D(a_5) = (x_4, x_2)$, $\psi_D(a_6) = (x_4, x_2)$,
  $\psi_D(a_7) = (x_5, x_2)$, $\psi_D(a_8) = (x_2, x_5)$, $\psi_D(a_9) = (x_3, x_5)$.

1.1 Graph and Graphical Presentation

- **Example 1.1.2:**

  $H = (V(H), E(H), \psi_H)$ is a digraph, where

  $V(H) = \{y_1, y_2, y_3, y_4, y_5\}$,

  $E(H) = \{b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9\}$

  and ψ_H is defined by $\psi_H(b_1) = (y_1, y_2)$, $\psi_H(b_2) = (y_3, y_2)$, $\psi_H(b_3) = (y_3, y_3)$,

  $\psi_H(b_4) = (y_4, y_3)$, $\psi_H(b_5) = (y_4, y_2)$, $\psi_H(b_6) = (y_4, y_2)$,

  $\psi_H(b_7) = (y_5, y_2)$, $\psi_H(b_8) = (y_2, y_5)$, $\psi_H(b_9) = (y_3, y_5)$.

- **Def:**
  - If $V \times V$ is a set of ordered pair (x, y)'s, then

    ① G is called **directed graph** (**digraph**).

    ② $e \in E$; **directed edge** (**arc**).

    ③ if $\psi(e) = (x, y)$: x is called the **tail** of e; y is called the **head** of e;

    e is called an **out-going edge** of x; **in-coming edge** of y.

1.1 Graph and Graphical Presentation

- **Def:**
  - **If $V \times V$ is a set of unordered pair $\{x, y\}$'s, then**

    ① G is called an undirected graph.

    ② Use xy or yx instead of $\{x, y\}$.

    ③ $e \in E$: undirected edges.

- **Example 1.1.3:**

  $G = (V(G), E(G), \psi_G)$ is an undirected graph, where

  $V(G) = \{z_1, z_2, z_3, z_4, z_5, z_6\}$,

  $E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}$

  and ψ_G is defined by $\psi_G(e_1) = z_1z_2$, $\psi_G(e_2) = z_1z_4$, $\psi_G(e_3) = z_1z_6$,

  $\psi_G(e_4) = z_2z_3$, $\psi_G(e_5) = z_3z_4$, $\psi_G(e_6) = z_3z_6$,

  $\psi_G(e_7) = z_2z_5$, $\psi_G(e_8) = z_4z_5$, $\psi_G(e_9) = z_5z_6$.

1.1 Graph and Graphical Presentation

- **Def:**
  - **loop**: ex: $\psi(e) = (x, x)$.
  - **parallel edges (multi-edges)**: ex: $\psi(e_1) = (x, y) = \psi(e_2)$
  - $E_G(x, y) = \{e \in E(G): \psi(e) = (x, y)\}$
  - $\mu_G(x, y) = |E_G(x, y)|$
  - $\mu(G) = \max\{\mu_G(x, y): \forall\, x, y \in V(G)\}$: the **multiplicity** of G.

- **ex: in Example 1.1.1:**

  $D = (V(D), E(D), \psi_D)$ is a digraph, where

  $V(D) = \{x_1, x_2, x_3, x_4, x_5\}$,

  $E(D) = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9\}$

  and ψ_D is defined by $\psi_D(a_1) = (x_1, x_2)$, $\psi_D(a_2) = (x_3, x_2)$, $\psi_D(a_3) = (x_3, x_3)$,

  $\psi_D(a_4) = (x_4, x_3)$, $\psi_D(a_5) = (x_4, x_2)$, $\psi_D(a_6) = (x_4, x_2)$,

  $\psi_D(a_7) = (x_5, x_2)$, $\psi_D(a_8) = (x_2, x_5)$, $\psi_D(a_9) = (x_3, x_5)$.

  a_5, a_6 are parallel edges,

  a_7, a_8 are not.

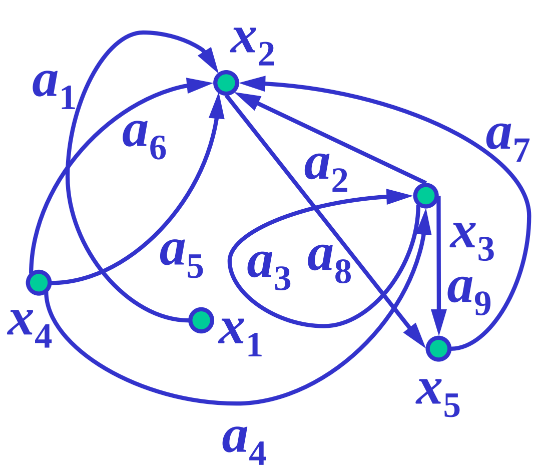
  a_3 is a loop.

1.1 Graph and Graphical Presentation

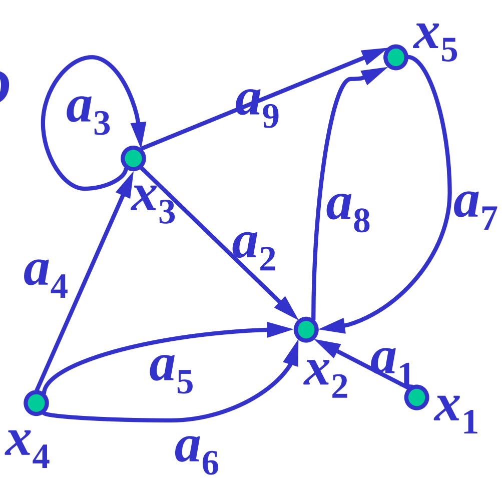
- **Def**: **graphical presentation**

- **ex**: in **Example 1.1.1**:
  $D = (V(D), E(D), \psi_D)$ is a digraph, where
  $V(D) = \{x_1, x_2, x_3, x_4, x_5\}$,
  $E(D) = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9\}$
  and ψ_D is defined by $\psi_D(a_1) = (x_1, x_2)$, $\psi_D(a_2) = (x_3, x_2)$, $\psi_D(a_3) = (x_3, x_3)$,
  $\psi_D(a_4) = (x_4, x_3)$, $\psi_D(a_5) = (x_4, x_2)$, $\psi_D(a_6) = (x_4, x_2)$,
  $\psi_D(a_7) = (x_5, x_2)$, $\psi_D(a_8) = (x_2, x_5)$, $\psi_D(a_9) = (x_3, x_5)$.

- **Figure 1.1,** (a) *D*



(b) *D*



1.1 Graph and Graphical Presentation

- <u>ex</u>: in <u>Example 1.1.3</u>:

  $G = (V(G), E(G), \psi_G)$ is an undirected graph, where

  $V(G) = \{z_1, z_2, z_3, z_4, z_5, z_6\}$,

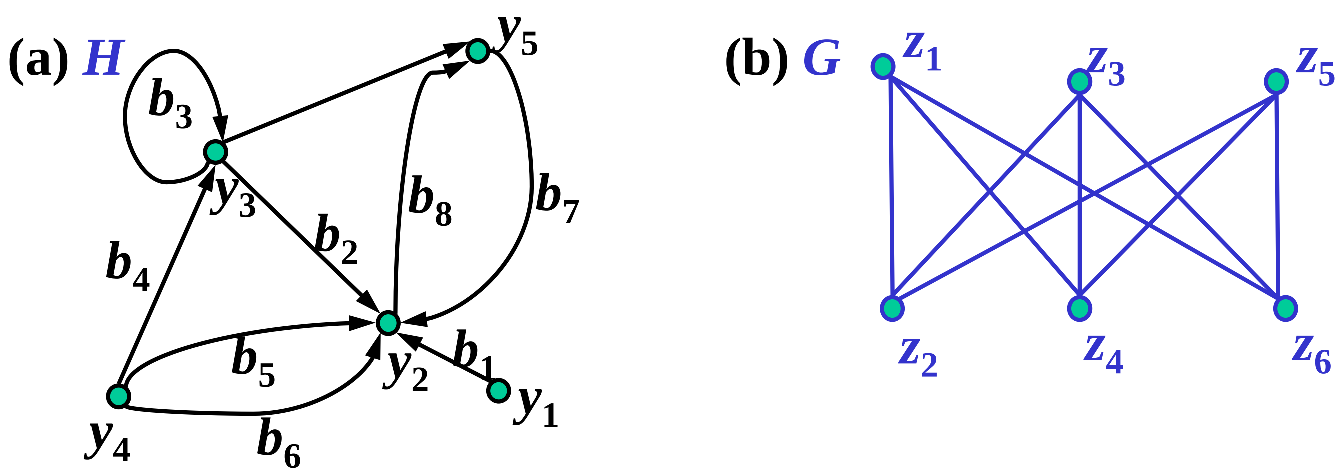
  $E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}$

  and ψ_G is defined by $\psi_G(e_1) = z_1z_2$, $\psi_G(e_2) = z_1z_4$, $\psi_G(e_3) = z_1z_6$,

  $\psi_G(e_4) = z_2z_3$, $\psi_G(e_5) = z_3z_4$, $\psi_G(e_6) = z_3z_6$,

  $\psi_G(e_7) = z_2z_5$, $\psi_G(e_8) = z_4z_5$, $\psi_G(e_9) = z_5z_6$.

- Figure 1.2

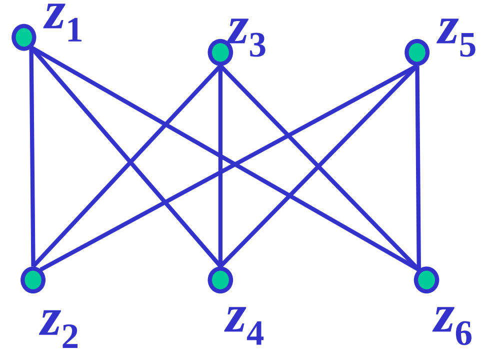


1.1 Graph and Graphical Presentation

- <u>**Def**</u>**:**
  - **incident: vertex ↔ edge.**
  - **adjacent: vertex ↔ vertex; edge ↔ edge.**
  - **loopless**
  - **simple: a graph contains neither loops nor parallel edges.**
    - **ψ is injective (1-1)**
    - **use $V \times V$ instead of E. i.e. write (V, E) for (V, E, ψ)**

- **ex: In <u>Example1.1.3</u>, write as : $G = (V(G), E(G))$, where**

  $E(G) = \{z_1z_2, z_1z_4, z_1z_6, z_2z_3, z_3z_4, z_3z_6, z_2z_5, z_4z_5, z_5z_6\}$.

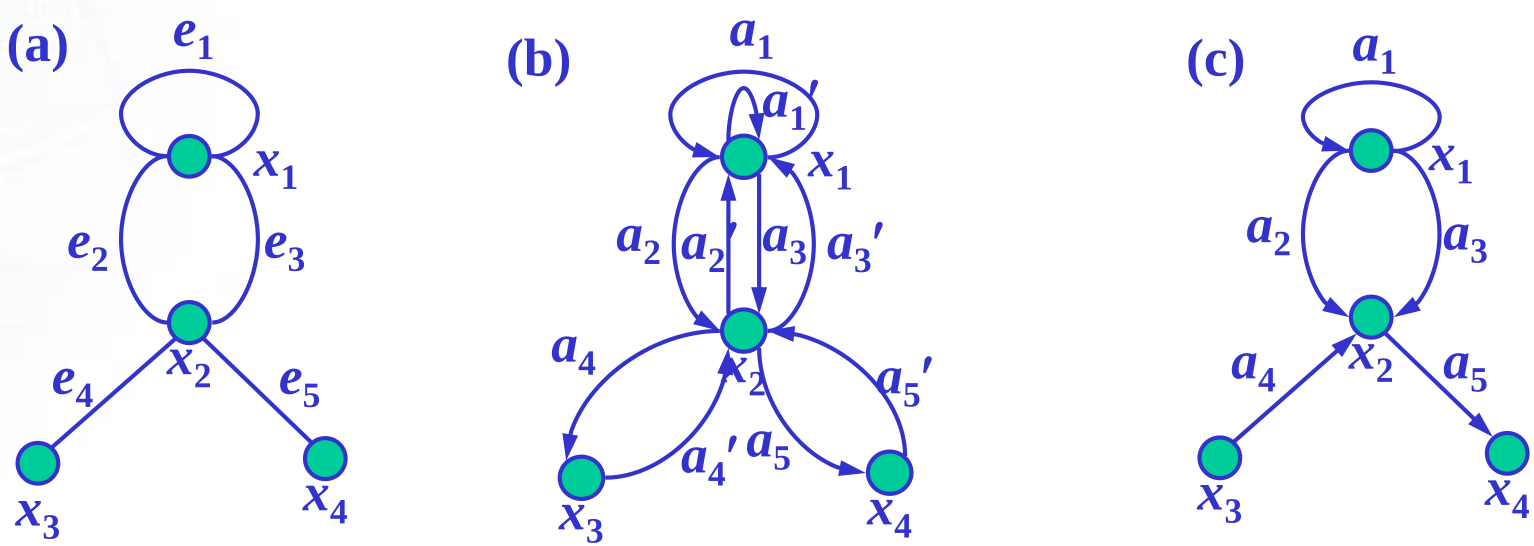


1.1 Graph and Graphical Presentation

- **Def:**
  - **An undirected graph can be thought of as a symmetric digraph, in which there are two symmetric edges, one in each direction, corresponding to each undirected edge.**
    $\rightarrow$ digraph is more general !!
  - **G is the underlying graph of $D \equiv D$ is an oriented graph of G.**

1.1 Graph and Graphical Presentation

- **ex: Figure 1.3 (a) an undirected graph,**
  (b) (a)'s symmetric digraph,
  (c) (a)'s an oriented graph.



1.1 Graph and Graphical Presentation

- **<u>Def</u>: Let $G = (V, E, \psi)$ be a graph,**
  - $\nu = |V|$ (= $n(G)$), is called **order** of G. (書上印成v)
  - $\varepsilon = |E|$ (= $m(G)$), is called **size** of G.
  - A graph is **empty** if $\varepsilon = 0$.
  - A empty graph is **trivial** if $\nu = 1$.
  - **non-trivial** ≡ not trivial.
  - A graph is **finite** if both ν, ε are finite. (本書只討論 finite graph)
  - $\lceil r \rceil$, $\lfloor r \rfloor$, $\binom{n}{k}$

1.1 Graph and Graphical Presentation

- **Example 1.1.4: In any group of six people, there must be three people who get to either know each other or not.**

  Proof.

  Let points *A*, *B*, *C*, *D*, *E*, *F* on the plane to denote these 6 people.

  Draw red line joining two points if they have known each other,

  blue line joining two points if not.

  Now, consider *F*, there exist three lines of the same color which are incident with *F*.

  (Without loss of generality) W.L.O.G.,

  suppose they are three red lines *FA*, *FB*, *FC*.

  Consider ΔABC, if it has no red line, then ∃ blue ΔABC,

  else it has a red line, say *AB*, then ∃ red ΔABF.

  That means, either ∃ three people (*ABF*) know each other,

  or ∃ three people (*ABC*) don't know each other.

1.1 Graph and Graphical Presentation

- **Exercise: 1.1.1, (剩下的請至少看過題目)**