Introduction

• 主要教科書：James A. Mchugh, Algorithmic Graph Theory, 開發, 1990.

• 重要參考書籍：
  2. Balakrishnan and Ranganatha, A Textbook of Graph Theory, Springer (俊傑代理), 2000
  3. Dieter Jungnickel, Networks and Algorithms, 俊傑, 1999
Introduction

• 課程內容：預計將介紹下列各項
  1. Introduction to Graph Theory
  2. Algorithmic Techniques
  3. Maximum Mathching Algorithm
  4. Shortest Paths
  5. Connectivity and Routing
  6. Graph Coloring
  7. Domination
  8. Covering
  9. NP-completeness

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Introduction

• 評量方式：
  期中考30% + 期末報告40% + 平時成績30% + 加分
  大學部及格60, 研究生及格70, 最高分99

• 進度：
  5/5 期中考: Chap 1 ~ Chap 5.
  6/16, 6/23 期末報告

• 網頁: http://www.csie.ncnu.edu.tw/~jsjuan/courses.html
Chapter 1
Introduction to Graph Theory

§ 1.1 Introduction to Graph Theory
1.1 Introduction to Graph Theory

- **Def:** A graph is an ordered pair $G = (V, E)$, where $V$ is a finite set of elements called vertices, and $E$ is a set of un-ordered pairs of distinct vertices, called edges.

- **Ex:** $V = \{a, b, c\}$, $E = \{\{a, b\}, \{b, c\}\}$

- **Note:** 有時以 “(a, b)” 或 “ab” 表示 “[a, b]”, 習慣上：$|V| = n$, $|E| = m$.

- **Def:**
  1. $x$ is adjacent to $y \equiv xy \in E$
  2. $x$ is incident to $\{a, b\} \equiv x = a$ or $x = b$
  3. $\{a, b\}$ is adjacent to $\{c, d\} \equiv \{a, b\} \neq \{c, d\} \land \{a, b\} \cap \{c, d\} \neq \emptyset$
1.1 Introduction to Graph Theory

• **Def:**

  ④ $\text{Adj}(x) = \{y \mid y \text{ is adjacent to } x\}$
  ⑤ $\text{Adj}(S) = \bigcup_{x \in S} \text{Adj}(x)$, if $S \subseteq V$
  ⑥ degree of a vertex $x \equiv \text{deg}(x) = |\text{Adj}(x)|$
  ⑦ matching $\equiv$ a set of edges in which no two distinct edges are adjacent.
  ⑧ A $(v_0, v_n)$ walk in a graph $G$ is an alternating sequence $W$:
    $v_0e_1v_1e_2v_2...e_nv_n$, where $v_i$’s are vertices and $e_i$’s are edges and $e_i$
    is incident to $v_{i-1}$ and $v_i$.
  ⑨ A path is a walk which all vertices are distinct.
  ⑩ A trial is a walk which all edges are distinct.
  ⑪ A cycle is a closed trial in which $v_0 = v_n$ and $v_1, ..., v_n$ are all distinct.
  ⑫ circuit = a closed trail.
1.1 Introduction to Graph Theory

• **Note:** $P$ is a path $\Rightarrow P$ is a trail.

• **Lemma:** $\forall$ graph $G = (V, E)$, $\sum_{v \in V} \deg(v) = 2|E|$.
Chapter 1
Introduction to Graph Theory

§ 1.2 Computer Representation of Graphs
1.2 Computer Representation of Graphs

- **Ex:** \( G = (V, E) \), \( V = \{a, b, c, d\} \). \( E = \{ab, ad, bc, bd, cd\} \)

\( V = \{1, 2, 3, 4\} \)

\[ \begin{array}{cccc}
1 & 0 & 1 & 0 \\
2 & 1 & 0 & 1 \\
3 & 0 & 1 & 0 \\
4 & 1 & 1 & 0 \\
\end{array} \]

\[ \begin{array}{cccc}
1 & 2 & \text{---} & 4 \\
2 & 1 & \text{---} & 3 \text{---} 4 \\
3 & 2 & \text{---} & 4 \\
4 & 1 & \text{---} & 2 \text{---} 3 \\
\end{array} \]

- **Note:** \( V \): 一般以\( \{1, 2, \ldots, n\} \)表之。

\( E : 1. \) Adjacency matrix: \( A = (a_{ij})_{n \times n} \), \( n = |V| \).

\[ a_{ij} = \text{# of edges corresponding to } \{v_i, v_j\}. \]

2. incident list: (adjacent list, edge list)
1.2 Computer Representation of Graphs

• **Note:**

  3. sequential adjacent list: (Columbic's Algorithmic Graph Theory and Perfect graphs)

• **Ex:** (前例)

```
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
```

```
Adj 2 4 1 3 4 2 4 1 2 3
```

```
Beg 1 3 6 8 11
```

<table>
<thead>
<tr>
<th>In paper:</th>
<th>In algorithm:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \forall j \in \text{Adj}(i) )</td>
<td>for ( \text{beg}(i) \leq k &lt; \text{beg}(i+1) )</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( j = \text{Adj}(k) );</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
</tbody>
</table>

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Chapter 1
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§ 1.3 Time Complexity
1.3 Time Complexity

- **Def:** $f(n) = O(g(n)) \iff \exists \ n_0 > 0, \ \exists \ c > 0$ 
  \[ \text{s.t. } f(n) \leq c \cdot g(n), \ \forall n \geq n_0. \]

- **Note:** $G = (V, E)$, **input:** $O(|V| + |E|) = O(|E|)$  
  **Sol.**
  
  ① matrix: $|V|^2$
  ② adj. list: $\sum_{v \in V} \deg(v) = 2|E| = O(|E|)$

  （分析時，以最佳者代入）