Computer Science and Information Engineering
National Chi Nan University

Discrete Mathematics

Dr. Justie Su-Tzu Juan

Chapter 2 Fundamentals of Logic

§ 2.1 Basic Connectives and Truth tables

Slides for a Course Based on the Text
Discrete & Combinatorial Mathematics (5th Edition)
by Ralph P. Grimaldi

§ 2.1 Basic Connectives and Truth tables

Def : ***statement*** **(*proposition*) : either *true* or *false*, not both.**

ex :
- ✓ ***p*: Combinatorics is a required course for sophomores.**
- ✓ ***q*: Margaret Mitchell wrote Gone with the Wind.**
- ✓ ***r*: 2 + 3 = 5.**
- ✗ **"What a beautiful evening!"**
- ✗ **"Get up and do your exercises."**
- ✗ **"The number *x* is an integer."**

Def : **1. *primitive statement*: No way to break them down into anything simpler.**

2. 反之: *compound statement*

§ 2.1 Basic Connectives and Truth tables

Def : 1. ***negation***; denoted by $\neg p$; read as "***not p***".

ex : 上ex中*p*:

$\neg p$ = "Combinatorics is not a required course for sophomores."

Def : 2. ***compound statement***, using the following ***logical connectives***.

a) ***Conjunction***; denoted by $p \wedge q$; read as "***p and q***".

b) ***Disjunction***;

$\begin{cases} \text{denoted by } p \vee q; \text{ read as "}p \text{ (inclusive) or } q\text{".} \\ \text{denoted by } p \underline{\vee} q; \text{ read as "}p \text{ exclusive or } q\text{".} \end{cases}$

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Def : 2. *compound statement*.

c) *Implication*; denoted by $p \to q$; read as "*p implies q*".

≡ (i) If *p*, then *q*

(ii) *p* is *sufficient* for *q*

(iii) *p* is a *sufficient condition* for *q*

(iv) *p* only if *q*

(v) *q* is *necessary* for *p*

(vi) *q* is a *necessary condition* for *p*

(vii) *p* is called the *hypothesis* of the implication.

(viii) *q* is called the *conclusion* of the implication.

d) *Biconditional*; denoted by $p \leftrightarrow q$; read as "*p if and only if q*".

(i) "*p* is *necessary and sufficient* for *q*."

(ii) "*p iff q*."

§ 2.1 Basic Connectives and Truth tables

Def : ***truth table***: **“0” for false and “1” for true.**

Table2.1:
2.2:

p	$\neg p$
0	1
1	0

p	q	$p \wedge q$	$p \vee q$	$p \veebar q$	$p \rightarrow q$	$p \leftrightarrow q$
0	0	0	0	0	1	1
0	1	0	1	1	1	0
1	0	0	1	1	0	0
1	1	1	1	0	1	1

ex : **“If 2 + 3 = 6, then 2 + 4 = 7”** **is true.**

§ 2.1 Basic Connectives and Truth tables

EX 2.1 : s: Phyllis goes out for a walk.

t: The moon is out.

u: It is snowing.

a) $(t \wedge \neg u) \rightarrow s$:

b) $t \rightarrow (\neg u \rightarrow s)$: "$\neg u \rightarrow s$" means "$(\neg u) \rightarrow s$", not "$\neg (u \rightarrow s)$"

c) $\neg (s \leftrightarrow (u \vee t))$:

d) "Phyllis will go out walking if and only if the moon is out" : $s \leftrightarrow t$

e) "If it is snowing and the moon is not out, then Phyllis will not go out for a walk" : $(u \wedge \neg t) \rightarrow \neg s$

f) "It is snowing but Phyllis will still go out for a walk" : $u \wedge s$

(where "but" ≡ "and")

§ 2.1 Basic Connectives and Truth tables

EX 2.2 : **"If I weigh more than 120 pounds, then I shall enroll in an exercise class".**

p: I weigh more than 120 pounds.

q: I shall enroll in an exercise class.

Penny's statement: $p \rightarrow q$

Case 1: $p = 1$ and $q = 1$: $>$ 120 pounds and enrolls :

Case 2: $p = 1$ and $q = 0$: $>$ 120 pounds but not enroll :

Case 3: $p = 0$ and $q = 0$: $\leq$ 120 pounds and not enroll :

Case 4: $p = 0$ and $q = 1$: $\leq$ 120 pounds but still enroll :

§ 2.1 Basic Connectives and Truth tables

EX 2.3 : In computer science: if-then, if-then-else.

ex : if $x > 2$ (執行時，給定 "x"值，則 "$x > 2$"為一 "logical statement")
then $y = 2$ ("executable statement", not "logical statement")
else $y = 3$ ("executable statement", not "logical statement")

ex : 生活上的 "$\rightarrow$" 與 "$\leftrightarrow$"

$s \rightarrow t$: **If you do your homework, then you will get to watch the baseball game.**

$t \rightarrow s$: **You will get to watch the baseball game only if you do your homework.**

§ 2.1 Basic Connectives and Truth tables

EX 2.4 : **"Margaret Mitchell wrote Gone with the Wind, and if $2 + 3 \neq 5$, then combinatorics is a required course for sophomores".**

$\equiv q \wedge (\neg r \rightarrow p)$

p	q	r	$\neg r$	$\neg r \rightarrow p$	$q \wedge (\neg r \rightarrow p)$
0	0	0	1	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	0	1	1
1	0	0	1	1	0
1	0	1	0	1	0
1	1	0	1	1	1
1	1	1	0	1	1

§ 2.1 Basic Connectives and Truth tables

EX 2.5 : **The truth tables for ① $p \vee (q \wedge r)$; ② $(p \vee q) \wedge r$.**

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$(p \vee q) \wedge r$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	1	1
1	0	0	0	1	1	0
1	0	1	0	1	1	1
1	1	0	0	1	1	0
1	1	1	1	1	1	1

∴不可以只寫$p \vee q \wedge r$，需標明為 $p \vee (q \wedge r)$ 或 $(p \vee q) \wedge r$!!

§ 2.1 Basic Connectives and Truth tables

EX 2.6 : $p \to (p \vee q), p \wedge (\neg p \wedge q)$

p	q	$p \vee q$	$p \to (p \vee q)$	$\neg p$	$\neg p \wedge q$	$p \wedge (\neg p \wedge q)$
0	0	0	1	1	0	0
0	1	1	1	1	1	0
1	0	1	1	0	0	0
1	1	1	1	0	0	0

$p \to (p \vee q)$ is true for all truth value;
$p \wedge (\neg p \wedge q)$ is false for all truth value.

§ 2.1 Basic Connectives and Truth tables

Def 2.1 : A compound statement is called a **contradiction** (**tautology**) if it is false (true) for all truth value assignments for its component statements, denoted by F_0 (T_0).

ex : $(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \rightarrow q$

only need to prove:

"when $p_1 = p_2 = \ldots = p_n = 1$ and q must $= 1$",

then $(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \rightarrow q$ is a tautology and we have a **valid argument**.

Def : Where such p_i is called **given statements** (**premises**); q is called **conclusion**.

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Chapter 2 Fundamentals of Logic

§ 2.2 Logical Equivalence: The Laws of Logic

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§ 2.2 Logical Equivalence: The Laws of Logic

In arithmetic and algebra: $x = y$ iff $|x| = |y|$ and $xy > 0$.
In geometry: $\triangle ABC \cong \triangle DEF$ iff $\overline{AB} = \overline{DE}$ and $\overline{BC} = \overline{EF}$ and $\overline{CA} = \overline{FD}$. In logical? algebra of propositions

Ex 2.7 :

p	q	$\neg p$	$\neg p \vee q$	$p \to q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	0	1	1

$\neg p \vee q \Leftrightarrow p \to q$

Def 2.2 : Two statements s_1, s_2 are said to be ***logically equivalent***, write $s_1 \Leftrightarrow s_2$, when

$$\begin{cases} s_1 \text{ is true iff } s_2 \text{ is true;} \\ s_1 \text{ is false iff } s_2 \text{ is false.} \end{cases}$$

§ 2.2 Logical Equivalence: The Laws of Logic

ex :

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$	$p \leftrightarrow q$
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	1	1	1	1

$(p \leftrightarrow q) \Leftrightarrow (p \to q) \wedge (q \to p) \quad \therefore (p \leftrightarrow q) \Leftrightarrow (\neg p \vee q) \wedge (\neg q \vee p)$

ex :

p	q	$p \veebar q$	$p \vee q$	$p \wedge q$	$\neg (p \wedge q)$	$(p \vee q) \wedge \neg (p \wedge q)$
0	0	0	0	0	1	0
0	1	1	1	0	1	1
1	0	1	1	0	1	1
1	1	0	1	1	0	0

$(p \veebar q) \Leftrightarrow (p \vee q) \wedge \neg (p \wedge q)$

§ 2.2 Logical Equivalence: The Laws of Logic

Note : **In fact, we may even eliminate either $\wedge$ or $\vee$.**

In real number, $\forall a, b \in \mathbb{R}$, $-(a+b) = (-a) + (-b)$.
In logical?

Ex 2.8 :

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$p \vee q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
0	0	0	1	1	1	1	0	1	1
0	1	0	1	1	0	1	1	0	0
1	0	0	1	0	1	1	1	0	0
1	1	1	0	0	0	0	1	0	0

DeMorgan's Laws : (i) $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$

(ii) $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

§ 2.2 Logical Equivalence: The Laws of Logic

In real number, $\forall$ *a*, *b*, *c* $\in \mathcal{R}$, $a \times (b + c) = (a \times b) + (a \times c)$
(Distributive Law of Multiplication over Addition)

Ex 2.9 :

p	q	r	$p\wedge(q\vee r)$	$(p\wedge q)\vee(p\wedge r)$	$p\vee(q\wedge r)$	$(p\vee q)\wedge(p\vee r)$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	1	1
1	0	0	0	0	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

The Distributive Laws of $\wedge$ over $\vee$: $p\wedge(q\vee r) \Leftrightarrow (p\wedge q)\vee(p\wedge r)$.

The Distributive Laws of $\vee$ over $\wedge$: $p\vee(q\wedge r) \Leftrightarrow (p\vee q)\wedge(p\vee r)$.

§ 2.2 Logical Equivalence: The Laws of Logic

Note : $a + (b \times c) \neq (a + b) \times (a + c)$

ex : $a = 2$, $b = 3$, $c = 5$, $a + (b \times c) = 17$, $(a + b) \times (a + c) = 35$.

Remark A :

1) When "($s_1 \leftrightarrow s_2$) is a tautology", then "$s_1 \Leftrightarrow s_2$".
   When "$s_1 \Leftrightarrow s_2$", then " ($s_1 \leftrightarrow s_2$) is a tautology".
2) If s_1, s_2, s_3 are statements, and $s_1 \Leftrightarrow s_2$ and $s_2 \Leftrightarrow s_3$, then
   $s_1 \Leftrightarrow s_3$.
3) $s_1 \not\Leftrightarrow s_2$ means s_1 and s_2 are not logically equivalent.

§ 2.2 Logical Equivalence: The Laws of Logic

The Laws of Logic (1/2)：	
1) $\neg\neg p \Leftrightarrow p$	**Law of Double Negation** (雙重否定律)
2) $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$ $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$	**DeMorgan's Laws** (迪摩根定律)
3) $p \vee q \Leftrightarrow q \vee p$ $p \wedge q \Leftrightarrow q \wedge p$	**Commutative Laws** (交換律)
4) $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$ $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$	**Associative Laws** (結合律)
5) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	**Distributive Laws** (分配律)

§ 2.2 Logical Equivalence: The Laws of Logic

The Laws of Logic (2/2)：

6) $p \vee p \Leftrightarrow p$ $p \wedge p \Leftrightarrow p$	**Idempotent Laws** (冪等律)
7) $p \vee F_0 \Leftrightarrow p$ $p \wedge T_0 \Leftrightarrow p$	**Identity Laws** (單一律、同一律)
8) $p \vee \neg p \Leftrightarrow T_0$ $p \wedge \neg p \Leftrightarrow F_0$	**Inverse Laws** (反定律、否定律)
9) $p \vee T_0 \Leftrightarrow T_0$ $p \wedge F_0 \Leftrightarrow F_0$	**Domination Laws** (支配律)
10) $p \vee (p \wedge q) \Leftrightarrow p$ $p \wedge (p \vee q) \Leftrightarrow p$	**Absorption Laws** (吸收律)

§ 2.2 Logical Equivalence: The Laws of Logic

Def 2.3 : s: statement contains only "$\neg$", "$\vee$", "$\wedge$", then the *dual* of s, denoted by s^d ≡ replacing each $\wedge$ and $\vee$ by $\vee$ and $\wedge$, respectively, and T_0 and F_0 by F_0 and T_0, respectively.

ex : $s = (p \wedge \neg q) \vee (r \wedge T_0)$, $s^d = (p \vee \neg q) \wedge (r \vee F_0)$.

Thm 2.1 : (*The Principle of Duality*) Let s and t be statements that contains no logical connectives other than $\wedge$ and $\vee$. If $s \Leftrightarrow t$, then $s^d \Leftrightarrow t^d$.

Corollary : Law 2 through 10 can be established by proving one of the laws in each pair.

§ 2.2 Logical Equivalence: The Laws of Logic

Substitution rules

ex : $(r \wedge s) \rightarrow q \Leftrightarrow \neg (r \wedge s) \vee q$ (see Table 2.11)

In Ex 2.7 : $\neg p \vee q \Leftrightarrow p \rightarrow q$

replace each "p" by "$r \wedge s$", get $(r \wedge s) \rightarrow q \Leftrightarrow \neg (r \wedge s) \vee q$, too.

Remark B : (*Substitution rules*)

1) The compound statement P is a tautology and p is a primitive statement in P: Replace **each** p by the **same** q, get P_1, then P_1 is also a tautology. **(S1)**

2) Let P be a compound statement and p is an arbitrary statement in P and let $q \Leftrightarrow p$: Replace **one or more** p by q get P_1, then $P_1 \Leftrightarrow P$. **(S2)**

§ 2.2 Logical Equivalence: The Laws of Logic

Ex 2.10 : a) P: $\neg (p \vee q) \leftrightarrow \neg p \wedge \neg q$ is a tautology.

replace each p by $r \wedge s$:

P_1: $\neg [(r \wedge s) \vee q] \leftrightarrow [\neg (r \wedge s) \wedge \neg q]$ is also a tautology.

replace each q by $t \rightarrow u$:

P_2: $\neg [(r \wedge s) \vee (t \rightarrow u)] \leftrightarrow [\neg (r \wedge s) \wedge \neg (t \rightarrow u)]$ is a tautology.

b)

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$[p \wedge (p \rightarrow q)] \rightarrow q$
0	0	1	0	1
0	1	1	0	1
1	0	0	0	1
1	1	1	1	1

$[p \wedge (p \rightarrow q)] \rightarrow q$ is a tautology.

replace each p by $r \rightarrow s$, q by $\neg t \vee u$:

$[(r \rightarrow s) \wedge [(r \rightarrow s) \rightarrow (\neg t \vee u)]] \rightarrow (\neg t \vee u)$ is a tautology.

§ 2.2 Logical Equivalence: The Laws of Logic

Ex 2.11 : a) Let P: $(p \to q) \to r$, $\because$ $(p \to q) \Leftrightarrow \neg p \vee q$

Let P_1: $(\neg p \vee q) \to r$, then $P_1 \Leftrightarrow P$,

i.e. $(p \to q) \to r \Leftrightarrow (\neg p \vee q) \to r$.

b) Let P: $p \to (p \vee q)$, $\because$ $\neg\neg p \Leftrightarrow p$

Let P_1: $p \to (\neg\neg p \vee q)$, then $P_1 \Leftrightarrow P$.

Let P_2: $\neg\neg p \to (\neg\neg p \vee q)$, then $P_2 \Leftrightarrow P$, too.

Ex 2.12 : Negate and simplify the com. statement $(p \vee q) \to r$.

Sol. 1) $(p \vee q) \to r \Leftrightarrow \neg (p \vee q) \vee r$ (by (S1) and $(s \to t) \Leftrightarrow \neg s \vee t$)

2) Negating: $\neg [(p \vee q) \to r] \Leftrightarrow \neg [\neg (p \vee q) \vee r]$ (by (S2))

3) $\neg [\neg (p \vee q) \vee r] \Leftrightarrow \neg\neg (p \vee q) \wedge \neg r$

(by DeMorgan's Law and (S1))

4) $\neg\neg (p \vee q) \wedge \neg r \Leftrightarrow (p \vee q) \wedge \neg r$

(by Law of Double Negation, (S1) and (S2))

$\therefore \neg [(p \vee q) \to r] \Leftrightarrow (p \vee q) \wedge \neg r$

§ 2.2 Logical Equivalence: The Laws of Logic

Ex 2.13 : p: Joan goes to Lake George.

q: Mary pays Joan's shopping spree

$p \rightarrow q$: If Joan goes to Lake George, then Mary pays for Joan's shopping spree.

$\neg(p \rightarrow q)$: ?

Sol.

$\because p \rightarrow q \Leftrightarrow \neg p \vee q$

$\therefore \neg(p \rightarrow q) \Leftrightarrow \neg(\neg p \vee q)$

$\Leftrightarrow \neg\neg p \wedge \neg q$ (by DeMorgan's Law)

$\Leftrightarrow p \wedge \neg q$ (by Law of Double Negation)

$\therefore \neg(p \rightarrow q)$: Joan goes to Lake George, but Mary does not pay for Joan's shopping spree.

§ 2.2 Logical Equivalence: The Laws of Logic

Note : **The negation of an if-then statement does not begin with the word if. ∵ It is not another implication.**

Ex 2.14 : $s: p \to q,\ s^d = ?$

Sol. $\because p \to q \Leftrightarrow \neg p \vee q \quad \therefore s^d = (\neg p \vee q)^d : \neg p \wedge q$

Ex 2.15 :

p	q	$p \to q$	$\neg q \to \neg p$	$q \to p$	$\neg p \to \neg q$
0	0	1	1	1	1
0	1	1	1	0	0
1	0	0	0	1	1
1	1	1	1	1	1

$(p \to q) \Leftrightarrow (\neg q \to \neg p);\ \ (q \to p) \Leftrightarrow (\neg p \to \neg q).$

§ 2.2 Logical Equivalence: The Laws of Logic

Def : 1) $\neg q \rightarrow \neg p$ is call the ***contrapositive*** of $p \rightarrow q$. (反證命題)

2) $q \rightarrow p$ is call the ***converse*** of $p \rightarrow q$. (逆命題)

3) $\neg p \rightarrow \neg q$ is call the ***inverse*** of $p \rightarrow q$. (轉命題)

Note : $(p \rightarrow q) \not\Leftrightarrow (q \rightarrow p)$; $(\neg p \rightarrow \neg q) \not\Leftrightarrow (\neg q \rightarrow \neg p)$

ex : p: Today is Mother's day.

q: Tomorrow is Monday.

- The implication $p \rightarrow q$: TRUE
- The contrapositive $\neg q \rightarrow \neg p$: TRUE
- The converse $q \rightarrow p$: ?
- The inverse $\neg p \rightarrow \neg q$: ?

§ 2.2 Logical Equivalence: The Laws of Logic

Ex 2.16 : **Find a simpler statement that is logically equivalent to $(p \vee q) \wedge \neg(\neg p \wedge q)$.**

Sol. (Not mention any application of (S1) (S2))

$(p \vee q) \wedge \neg(\neg p \wedge q)$

$\Leftrightarrow (p \vee q) \wedge (\neg\neg p \vee \neg q)$ (∵ DeMorgan's Law)

$\Leftrightarrow (p \vee q) \wedge (p \vee \neg q)$ (∵ Law of Double Negation)

$\Leftrightarrow p \vee (q \wedge \neg q)$ (∵ Distributive Law of $\vee$ over $\wedge$)

$\Leftrightarrow p \vee F_0$ (∵ Inverse Law)

$\Leftrightarrow p$ (∵ Identity Law)

$\therefore (p \vee q) \wedge \neg(\neg p \wedge q) \Leftrightarrow p$

Ex: "這周離散和計概至少有一堂課會點名，並且不可能離散不點名而計概點名"

§ 2.2 Logical Equivalence: The Laws of Logic

Ex 2.17 : **Find a simpler statement: $\neg[\neg[(p \vee q) \wedge r] \vee \neg q]$.**

Sol.

$\neg[\neg[(p \vee q) \wedge r] \vee \neg q]$

$\Leftrightarrow \neg\neg[(p \vee q) \wedge r] \wedge \neg\neg q$ (∵ DeMorgan's Law)

$\Leftrightarrow [(p \vee q) \wedge r] \wedge q$ (∵ Law of Double Negation)

$\Leftrightarrow (p \vee q) \wedge (r \wedge q)$ (∵ Associative Law of $\wedge$)

$\Leftrightarrow (p \vee q) \wedge (q \wedge r)$ (∵ Commutative Law of $\wedge$)

$\Leftrightarrow [(p \vee q) \wedge q] \wedge r$ (∵ Associative Law of $\wedge$)

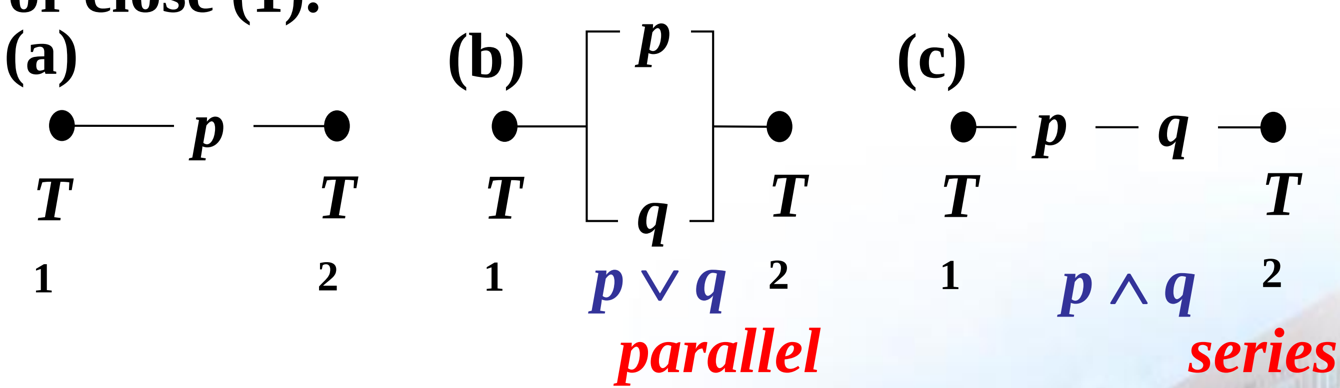
$\Leftrightarrow q \wedge r$ (∵ Absorption Law)

Ex: "今天沒有做到沒有暨遲到或早退且上課打瞌睡，或者沒有遲到。"

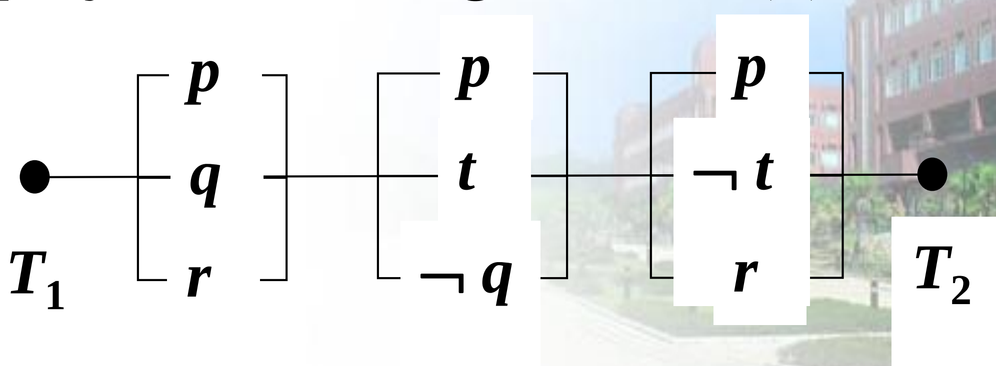
Note : **By Ex 2.7 : $\neg[[(p \vee q) \wedge r] \rightarrow \neg q] \Leftrightarrow \neg[\neg[(p \vee q) \wedge r] \vee \neg q]$. ∴ $\neg[[(p \vee q) \wedge r] \rightarrow \neg q] \Leftrightarrow q \wedge r$.**

§ 2.2 Logical Equivalence: The Laws of Logic

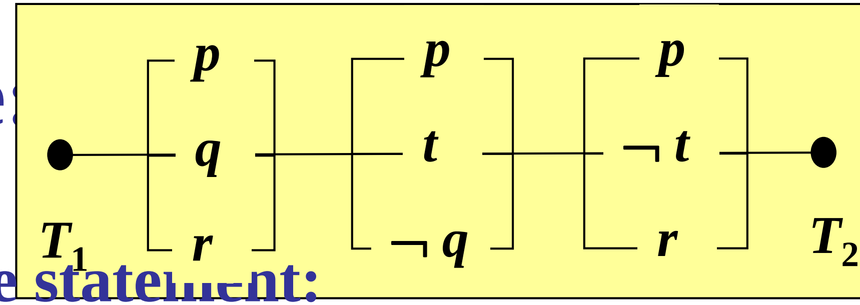
Ex 2.18 (1/3): Switching Network : wires, switches connecting two terminals T_1, T_2, each switch is either open (0) or close (1).



Simplify the switching network (d):



§ 2.2 Logical Equivalence:



Ex 2.18 (2/3): Represented by the statement:

$(p \vee q \vee r) \wedge (p \vee t \vee \neg q) \wedge (p \vee \neg t \vee r)$

$\Leftrightarrow p \vee [(q \vee r) \wedge (t \vee \neg q) \wedge (\neg t \vee r)]$ **(Distributive Law of ∨ over ∧)**

$\Leftrightarrow p \vee [(q \vee r) \wedge (\neg t \vee r) \wedge (t \vee \neg q)]$ **(Commutative Law of ∧)**

$\Leftrightarrow p \vee [((q \wedge \neg t) \vee r) \wedge (t \vee \neg q)]$ **(Distributive Law of ∨ over ∧)**

$\Leftrightarrow p \vee [((q \wedge \neg t) \vee r) \wedge (\neg \neg t \vee \neg q)$ **(Law of Double Negation)**

$\Leftrightarrow p \vee [((q \wedge \neg t) \vee r) \wedge \neg(\neg t \wedge q)]$ **(DeMorgan's Law)**

$\Leftrightarrow p \vee [\neg(\neg t \wedge q) \wedge ((\neg t \wedge q) \vee r)\ ]$ **(Commutative Law of ∧)**

$\Leftrightarrow p \vee [[\neg(\neg t \wedge q) \wedge (\neg t \wedge q)] \vee [\neg(\neg t \wedge q) \wedge r]\ ]$

(Distributive Law of ∧ over ∨)

$\Leftrightarrow p \vee [F_0 \vee [\neg(\neg t \wedge q) \wedge r]\ ]$ **(Inverse Law of ∧)**

$\Leftrightarrow p \vee [[\neg(\neg t \wedge q)] \wedge r]$ **(Identity Law of ∨)**

$\Leftrightarrow p \vee [r \wedge [\neg(\neg t \wedge q)]]$ **(Commutative Law of ∧)**

$\Leftrightarrow p \vee [r \wedge (t \vee \neg q)]$ **(DeMorgan's Law & Law of Double Negation)**

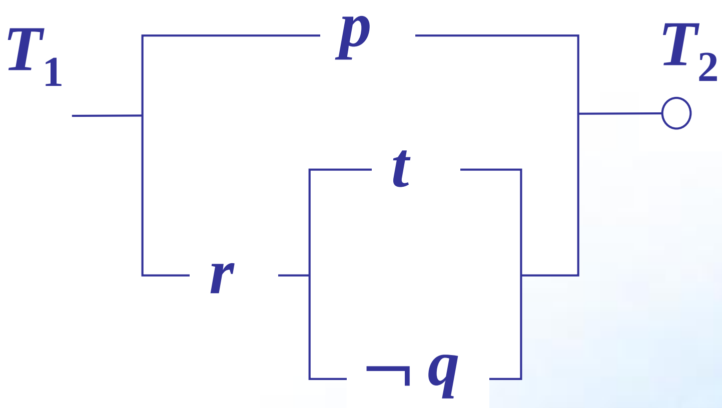
§ 2.2 Logical Equivalence: The Laws of Logic

Ex 2.18 (3/3):

Hence $(p \vee q \vee r) \wedge (p \vee t \vee \neg q) \wedge (p \vee \neg t \vee r)$

$\Leftrightarrow p \vee [r \wedge (t \vee \neg q)]$

network (e) :



∴ **the network (e) is equivalent to the original network (d)**