Computer Science and Information Engineering
National Chi Nan University

Discrete Mathematics

Dr. Justie Su-Tzu Juan

Chapter 0 Introduction

Slides for a Course Based on the Text
Discrete & Combinatorial Mathematics **(5th Edition)**
by Ralph P. Grimaldi

“離散數學"課程
核心能力與課程地圖

(c) Spring 2025, Justie Su-Tzu Juan

國立暨南國際大學
National Chi Nan University
行政大樓

暨大學生八大基本素養與核心能力

(一) 道德思辨與實踐能力

(二) 人際溝通與表達能力

(三) 獨立思考與創新能力

(四) 人文關懷與藝術涵養

(五) 專業知能與數位能力

(六) 團隊合作與樂業倫理

(七) 全球視野與尊重多元文化

(八) 社區參與與公民責任

Discrete Mathematics（離散數學）

Office : 科三422 #4875

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上課時間 / 地點 : 三BCD / 科三113

A. 課程目標：離散數學為一資工系學生所須俱備基本知識之一，將介紹基本計數、邏輯、集合、函數、關係等數學觀念及演算技巧，作為未來考試、就業或研究之基礎。

B. 主要教科書 ：

Grimaldi, Discrete and Combinatorial Mathematics 5/e, Addison-Wesley (新月代理), 2003

C. 重要參考書籍 ：

1. Feil and Krone, Essential Discrete Mathematics for Computer Science, Prentice-Hall (全華代理), 2003.
2. Grassmann and Trembley, Logic and Discrete Mathematics - a Computer Science Presoective, Prentice-Hall (全華代理), 1996
3. Liu, Elements of Discrete Mathematics 2/e, McGraw-Hill, 1998
4. Richard Johnsonbaugh, Discrete Mathematics 5/e, Prentice Hall, 2001
5. Rosen, Discrete Mathmatics and Its Applications, 6/e, McRrawW-HILL (歐亞代理→新月代理→?), 2007

D. 課程內容：

1. Fundamental Principles of Counting
2. Fundamentals of Logic
3. Set Theory
4. Properties of the Integers: Mathematical Induction
5. Relations and Functions
6. Languages: Finite State Machines
7. Relations: The Second Time Around

E. 計分方式: 最高分99

預計將有兩次期中考與一次期末考。

期中考2*20% + 期末考25% + 作業成績25%
+ 平時成績10% (最高分99)

F. 期中考: 3/26, 5/7

G. 期末考: 6/4

H. 作業：題號除六餘一者

作業於上課前繳交, 有疑問可於TA Time時詢問助教

I. 助教: 計算理論研究室 R307-1 (分機4862)

Office Hour :

二 13:10 ~ 14:00 周芊嘉

J. 分組

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Chapter 1 Fundamental Principles of Counting

§ 1.1 The Rules of Sum and Product

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§ 1.1 The Rules of Sum and Product

Outline:

1. **The Rule of Sum**
2. **The Rule of Product**
3. **Combine the Rules of Both Sum and Product**

§ 1.1 The Rules of Sum and Product

The Rule of Sum：**If a first task can performed in *m* ways, while a second task can be performed in *n* ways, and the two tasks cannot be performed simultaneously, then performed either task can be accomplished in any one of *m* + *n* ways.**

EX 1.1 : A college library has 40 textbooks on sociology and 50 textbooks dealing with anthropology. By the rule of sum, the college can select among 40 + 50 = 90 textbooks in order to learn more about one or the other of these two subjects.

§ 1.1 The Rules of Sum and Product

The rule can be extended beyond two tasks as long as no pair of the tasks can occur simultaneously.

<u>EX 1.2</u> : A computer science instructor who has, say, five introductory books each on C++, FORTRAN, Java and Pascal can recommend any one of there 20 books to a student who is interested in learning a first programming language.

$(5 + 5 + 5 + 5 = 20)$

§ 1.1 The Rules of Sum and Product

***The Rule of Product*：If a procedure can be broken down into first and second stages, and if there are *m* possible outcomes for the first stage and if, for each of these outcomes, there are *n* possible outcomes for the second stage, then the total procedure can be carried out, in the designated order, in *mn* ways.**

EX 1.3 : The drama club of Central University is holding tryouts for a spring play. With six men and eight women auditioning for the leading male and female roles, by the rule of product the director can cast his leading couple in $6 \times 8 = 48$ ways.

§ 1.1 The Rules of Sum and Product

Ex 1.6 : Here various extensions of the rule illustrated by considering the manufacture of license plates consisting of two letters followed by four digits.

a) If no letter or digit can be repeated.

b) With repetitions of letters and digits allowed.

c) If repetitions are allowed, as in part (b), how many of the plates have only vowels (A, E, I, O, U) and even digits? (0 is an even integer.)

Sol.

(a) $26 \times 25 \times 10 \times 9 \times 8 \times 7 = 3276000.$

(b) $26^2 \times 10^4 = 6760000.$

(c) $5^2 \times 5^4 = 5^6 = 15625.$

§ 1.1 The Rules of Sum and Product

EX 1.7 : **bit, byte, two-byte, 32-bit.**

Sol.

Bit : $0/1 \rightarrow 2$;

Byte : 8 bits $\rightarrow 2^8 = 256$;

Two-byte : 2×8 bits $\rightarrow 2^8 \times 2^8 = 2^{16} = 65536$;

32-bit : 4×8 bits $\rightarrow 2^8 \times 2^8 \times 2^8 \times 2^8 = 2^{32} = 4294967296$.

Combine the rules of both sum and product.

§ 1.1 The Rules of Sum and Product

EX 1.8 : At the AWL corporation Mrs. Forster operates the Quick Snack Coffee Shop. The menu at her shop is limited: six kinds of muffins, eight kinds of sandwiches, and five beverages (hot coffee, hot tea, iced tea, cola, and orange juice). Ms. Dodd, an editor at AWL, sends her assistant Carl to the shop to get her lunch — either a muffin and a hot beverage or a sandwich and a cold beverage. How many way in which Carl can purchase Ms. Dodd's lunch?

Sol.

By the rule of product :

muffin and hot beverage : $6 \times 2 = 12$ ways,

sandwich and cold beverage : $8 \times 3 = 24$ ways.

By the rule of sum :

$12 + 24 = 36$ ways : Carl can purchase Ms. Dodd's lunch.

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Chapter 1 Fundamental Principles of Counting

§ 1.2 Permutations

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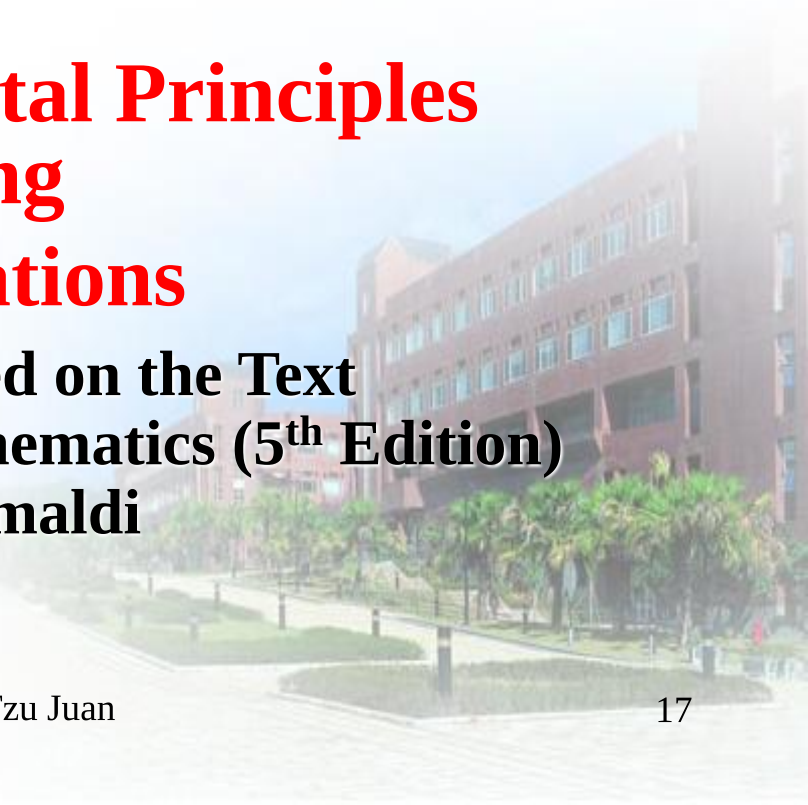


§ 1.2 Permutations

Outline:

1. **Definition:** ***n factorial***
2. **Permutation:**
   - □ **Distinct Objects**
   - □ **Not All Distinct Objects**
   - □ **No longer linear**

§ 1.2 Permutations

Def 1.1 : For an integer n, *n factorial* (denoted $n!$) is defined by $0! = 1$,

$$n! = (n)(n-1)(n-2)\ldots(3)(2)(1), \text{ for } n \geq 1.$$

Q : How fast the values of $n!$ increase :

10! seconds in six weeks;

11! > seconds in one year;

12! > seconds in 12 years;

13! > seconds in a century.

Def 1.2 : Given a collection of n distinct objects, any (linear) arrangement of there objects is called a *permutation* of the collection.

§ 1.2 Permutations

Distinct objects

If there are n distinct objects, denoted $a_1, a_2, \ldots, a_n$, and r is an integer, with $1 \le r \le n$, then by the rule of product, the number of permutations of size r for the n objects is

$$\underbrace{n}_{\text{1st position}} \times \underbrace{(n-1)}_{\text{2nd position}} \times \underbrace{(n-2)}_{\text{3rd position}} \times \ldots \times \underbrace{(n-r+1)}_{r\text{th position}} =$$

$$(n)(n-1)(n-2)\cdots(n-r+1)\frac{(n-r)(n-r-1)\cdots(3)(2)(1)}{(n-r)(n-r-1)\cdots(3)(2)(1)} = \frac{n!}{(n-r)!}$$

Denote this number by $P(n, r)$:

<u>Def :</u> 1. For $r = 0$, $P(n, 0) = 1 = n!/(n-0)!$

2. For all $1 \le r \le n$, $P(n, r) = n!/(n-r)!$.

§ 1.2 Permutations

EX 1.10 :

a) The number of permutations of the letters in the word COMPUTER is ?

Sol. $8!$.

b) If only five of the letters are used, the number of permutations (of size 5) is ?

Sol. $P(8, 5) = 8! / 3! = 6720$.

c) If repetitions of letters are allowed, the number of possible 12-letter sequences is ?

Sol. $8^{12} \cong 6.87 \times 10^{10}$.

§ 1.2 Permutations

Not all distinct objects

EX 1.11 : **The number of（linear）arrangements of the four letters in BALL is ?**

Sol. (1/2)

A B L L	$A\ B\ L_1\ L_2$	$A\ B\ L_2\ L_1$
A L B L	$A\ L_1\ B\ L_2$	$A\ L_2\ B\ L_1$
A L L B	$A\ L_1\ L_2\ B$	$A\ L_2\ L_1\ B$
B A L L	$B\ A\ L_1\ L_2$	$B\ A\ L_2\ L_1$
B L A L	$B\ L_1\ A\ L_2$	$B\ L_2\ A\ L_1$
B L L A	$B\ L_1\ L_2\ A$	$B\ L_2\ L_1\ A$
L A B L	$L_1\ A\ B\ L_2$	$L_2\ A\ B\ L_1$
L A L B	$L_1\ A\ L_2\ B$	$L_2\ A\ L_1\ B$
L B A L	$L_1\ B\ A\ L_2$	$L_2\ B\ A\ L_1$
L B L A	$L_1\ B\ L_2\ A$	$L_2\ B\ L_1\ A$
L L A B	$L_1\ L_2\ A\ B$	$L_2\ L_1\ A\ B$
L L B A	$L_1\ L_2\ B\ A$	$L_2\ L_1\ B\ A$

§ 1.2 Permutations

Sol. (2/2)

Let number(#) of arrangements of the letters B, A, L, L = X;
and number of permutations of the symbols B, A, L_1, L_2 = Y = 4!

$$2 \times X = Y \Rightarrow X = 4!/2 = 12.$$

EX 1.12 : What is the number of arrangements of all six letters in PEPPER?

Sol.

PEPPER $\Rightarrow$ $P_1EP_2P_3ER$, $P_1EP_3P_2ER$, $P_2EP_1P_3ER$, $P_2EP_3P_1ER$, $P_3EP_1P_2ER$, $P_3EP_2P_1ER$.

$P_1EP_2P_3ER$ $\Rightarrow$ $P_1E_1P_2P_3E_2R$, $P_1E_2P_2P_3E_1R$.

Let number of arrangements of the letters PEPPER = X;
and number of permutations of the symbols

$$P_1E_1P_2P_3E_2R = Y = 6!$$

$$(2!)(3!) \times X = Y \Rightarrow X = 6!/(2!3!) = 60.$$

§ 1.2 Permutations

If there are n objects with n_1 of a first type, n_2 of a second types,…, and n_r of an rth types, where $n_1 + n_2 + … + n_r = n$, then there are

$$\frac{n!}{n_1!n_2! \ldots n_r!}$$

(linear) arrangements of the n objects. (Objects of the same type are indistinguishable.)

<u>EX 1.13</u> : (a) Arranging all of the letters in MASSASAUGA.
How many is the possible arrangements?
(b) If all four A's are together?

Sol.

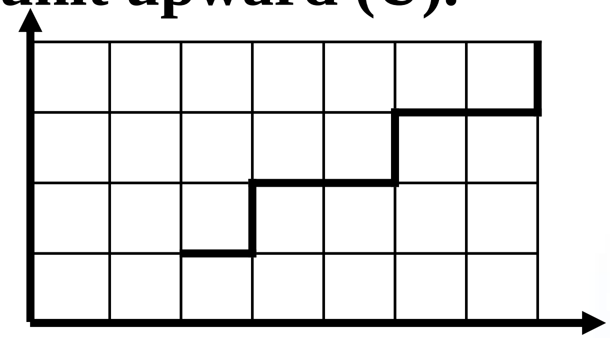
(a) $\dfrac{10!}{4!\,3!} = 25200$ (b) $\dfrac{7!}{3!} = 840$

§ 1.2 Permutations

<u>EX 1.14</u> : **Determine the number of (staircase) paths in the *xy*-plane from (2,1) to (7,4), where each such path is made up of individual steps going one unit to the right (R) or one unit upward (U).**

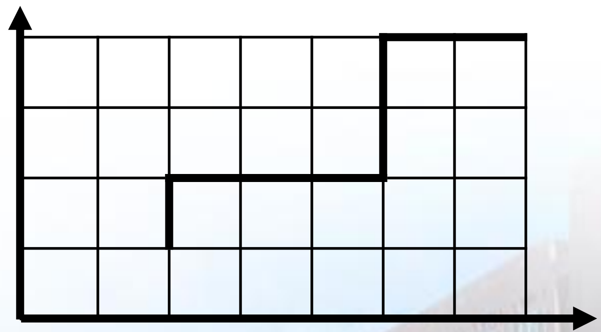
Sol.

(a)



R,U,R,R,U,R,R,U

(b)



U,R,R,R,U,U,R,R

From (2,1) to (7,4) requires 7 – 2 = 5 horizontal moves to the right and 4 – 1 = 3 vertical moves upward.

the # of paths = the # of arrangements of five R's and three U's.

= 8!/(5!3!) = 56.

§ 1.2 Permutations

<u>EX 1.5</u> : Prove that if n and k are positive integers with $n = 2k$, then $n!/2^k$ is an integer.

Proof.

Consider the n symbols $x_1, x_1, x_2, x_2, \ldots, x_k, x_k$.

The number of ways in which we can arrange all of there

$n = 2k$ symbols is an integer that equals

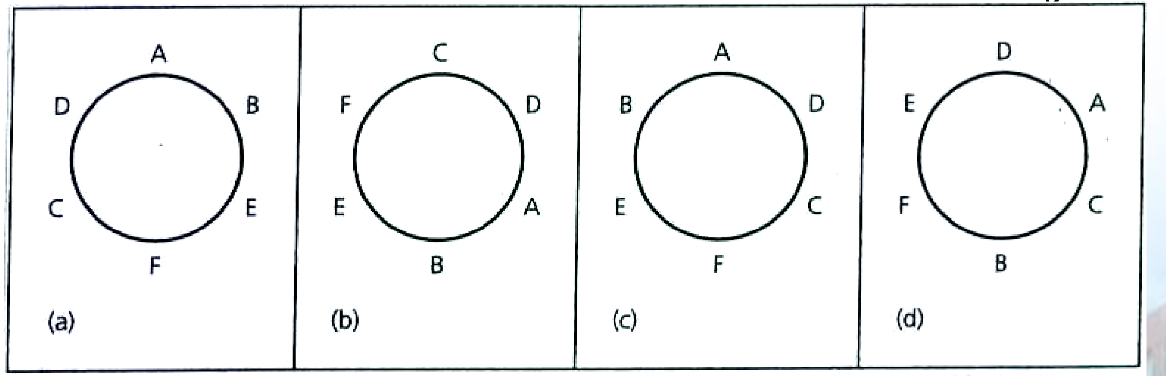
$$\frac{n!}{\underbrace{2!\,2!\,\ldots\,2!}_{k}} = \frac{n!}{2^k}.$$

<u>*No longer linear*</u>

§ 1.2 Permutations

<u>EX 1.16</u> : If six peoples, designated as A, B,…,F, are seated about a round table, how many different circular arrangements are possible, if arrangements are considered the same when one can be obtained from the other by rotation?

Sol.



ABEFCD $\Rightarrow$ BEFCDA, EFCDAB, FCDABE, CDABEF, DABEFC.

Let number of circular arrangements of A, B,…, F = X;
and number of linear arrangement of A, B,…, F = Y = 6!

$6 \times X = Y \Rightarrow X = 6!/6 = 120.$

§ 1.2 Permutations

EX 1.17 : Suppose now that the six people of EX1.16 are three married couples and that A, B, and C are females. We want to arrange the six people around the table so that sexes alternate. (Once again, arrangements are considered identical if one can be obtained from the other by rotation.)

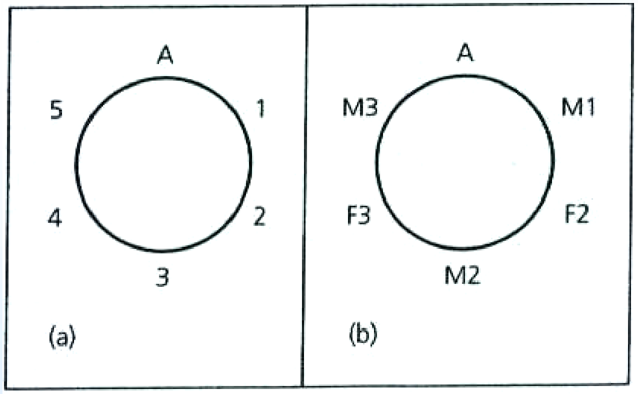
Sol.

A (a female) $\rightarrow$M1 $\rightarrow$ F2 $\rightarrow$ M2…

$\Rightarrow 3 \times 2 \times 2 \times 1 \times 1 = 12.$

Or : A, B, C $\rightarrow$M1, M2, M3

$\Rightarrow (3! / 3) \times 3! = 2 \times 6 = 12.$



Another method to solve EX 1.16 :

First place A, then filling there remain locations is the problem of permuting B, C, D, E, F in a linear manner.

$\Rightarrow 5! = 120.$

§ 1.2 Permutations

Checklist:

1. **Definition**
   - *n factorial* (denoted $n!$)
   - *permutation*
2. **Distinct Objects**
   - $P(n, r)$
3. **Not All Distinct Objects**
   - $\dfrac{n!}{n_1!n_2! \ldots n_r!}$
4. **No longer linear**
   - $(n-1)!$